Module-1 : Network Topology

Syllabus:

* Introduction & basic definitions of elementary graph theory
* tree, cut-set, loop.
* Formation of Incidence Matrices
* Primitive n/w impedance form & admittance form.
* Formation of Y bus by Singular transformation
* Y bus by Inspection method.
* Illustrative examples.

1.1 General introduction:

* In PSA-2 we deal with mathematical modelling & analysis of large electrical power n/ws.
* M1 – Represents a Complex power s/s as a n/w of buses & branches
  (Analogy) – Google maps shows a city as a s/s of roads & junctions.
* Practically – Every Generating station, transformer substation & load centre in the power grid acts as – Bus or Node.
  TLs or Cables acts as Branches (Links) b/w them
* Bus Incidence Matrix (A): To show how elements connect to bus.
* Bus Admittance matrix (Ybus) – To understand how currents & Vtgs flow in n/w
* In real life; there are essential due for
  → Power flow studies (what happens when load ↑ses)
  → Fault analysis (what happens during short ckts)
  → Stability checks (Can the grid survive a disturbance)
* Engineers in Control centers monitor & control the power flow using these Models.
* During real time operation, Softwares like SCADA uses Ybus & power flow Eqns which helps to predict overloading & plan to prevent blackouts. (1)

1.2 Elementary Graph Theory:

* Graph: The geometrical interconnection of the various branches is called topology of the n/w. The Connection of n/w topology, shown by replacing all physical elements by lines is called Graph

(or)

A graph shows the geometrical interconnection of the elements of a n/w

* A subgraph is any subset of elements of the graph.

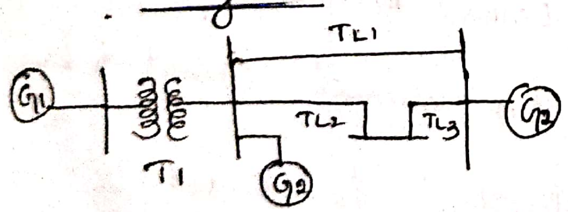
(or) A Subgraph is a smaller part of a larger graph

* Path: A path is a sequence of lines & nodes that connect ~~two~~ two buses without repeating any node

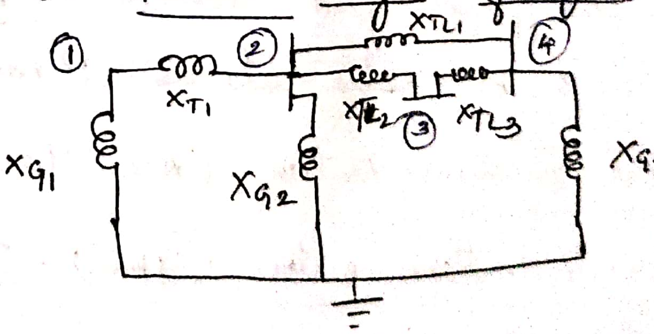
* Oriented graph: If each element of connected graph is assigned a direction then it is Oriented graph.

1.3 Consider the following: ?

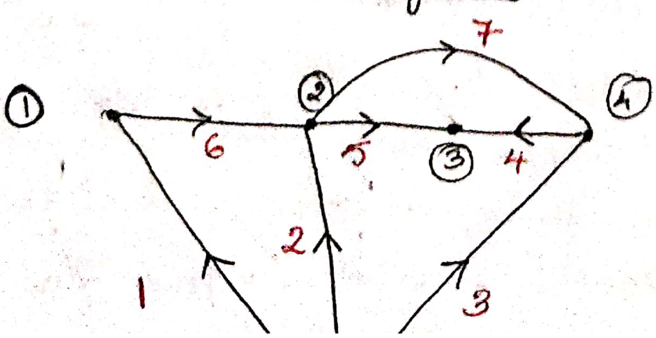
(1) Single line diagram



(2) Reactance diagram of fig ①



(3) Oriented graph



Let,

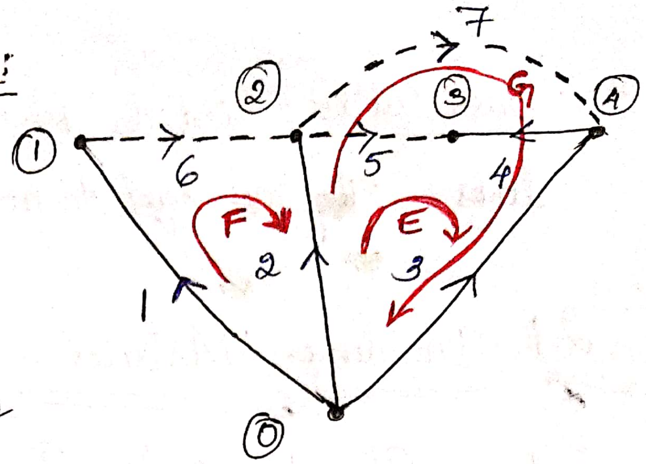
n - no. of nodes = 5

e - no. of lines or elements = 7

b - no. of branches

l - no. of links

1.4 Tree & Co-tree of the Oriented graph:



* Tree: A connected subgraph containing all nodes of a graph but no closed path.

* Branch: The elements of a tree are called branches & form a subset of the elements of the connected graph.

We have $\boxed{b = n - 1}$; where n - no. of nodes $= 5$

$b = 5 - 1 = 4$ ⇒ Branches are 1, 2, 3, 4 (solid lines)

* Links: Those elements of the connected graph that are not included in the tree are called links & forms a subgraph.

We have $\boxed{l = e - b}$ or $l = e - (n-1)$ or $\boxed{l = e - n + 1}$

where e - no. of elements.

⇒ $l = 7 - 4 = 3$

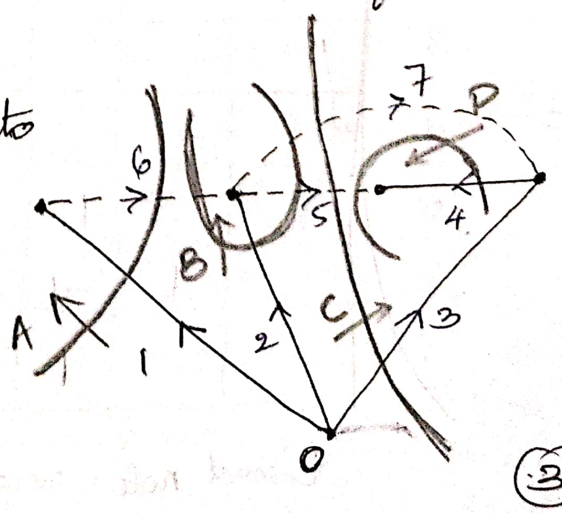
Links are 5, 6, 7 (dotted lines)

* Co-tree: It is compliment of tree. Links & co-tree are same.

* Loop: It is a closed path in a graph. It starts & ends at the same node. If a link is added to the tree, the resulting graph contains one closed path.

* Basic loop: The loops which contain only one link are independent & are called basic loops.
  * No. of loops = No. of links
  * Orientation of a basic loop is chosen - same as that of link

* Cut-Set: It is a set of elements that, if removed, divides a connected graph into two connected subgraphs



* Twig: Elements of a tree are called twigs or tree-branches

* **Basic cut set:** Cut sets which contain only one twig & remaining links. They are equal to no. of twigs.

2. Incidence Matrices:

2.1 Element - Node Incidence Matrix or Branch node incidence matrix or bus incidence matrix '$\hat{A}$' $=(e \times n)$

* The incidence of elements to nodes in a connected graph is shown by element-node incidence matrix
* The elements are obtained as follows.
  e - are placed on rows ; n - are placed on columns in matrix.

$\boxed{a_{ij} = 1}$ → If i^{th} element is incident to & oriented away from node

$\boxed{a_{ij} = -1}$ → If i^{th} element is incident to & oriented towards the node.

$\boxed{a_{ij} = 0}$ → If i^{th} element is not incident to j^{th} node.

* Dimension of matrix is $\boxed{\hat{A} = e \times n}$
* $\hat{A}$ for for oriented graph in 1.3 is

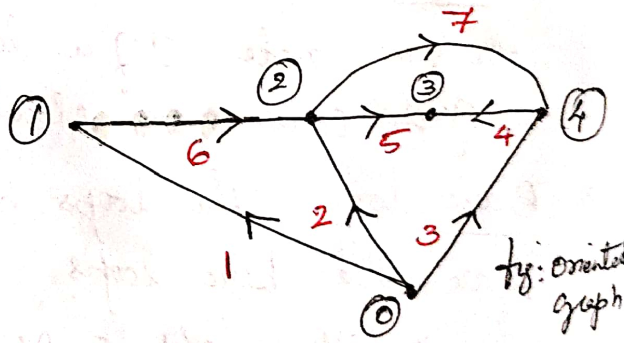


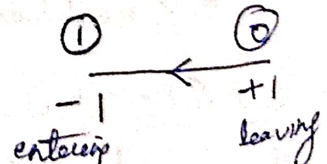
fig: oriented graph

Nodes →

$\hat{A} =$

e \ n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1				-1
4				-1	1
5			1	-1	
6		1	-1		
7			1		-1

elements ↓



(2)

$$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Element node incidence matrix - '$\hat{A}$'

* Any nodes of connected graph can be chosen as reference node & the column corresponding to that node is deleted from $\hat{A}$. The matrix so formed is called the Bus-incidence Matrix A.

* Dimension of $A = e \times (n-1)$; where $(n-1)$ – rank of matrix.

* From above example consider the node ⓪ as reference node. We get matrix A given by,

$$A = \begin{array}{c|cccc} e \backslash n & ① & ② & ③ & ④ \\ \hline 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ \hline 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & 1 \end{array} = \begin{array}{c} \text{Branches} \\ \& \\ \text{Links} \end{array} \begin{bmatrix} A_b \\ \hline A_l \end{bmatrix}$$

* Above matrix is partitioned into 2 submatrices as shown above:

$A_b = b \times (n-1)$

$A_l = l \times (n-1)$

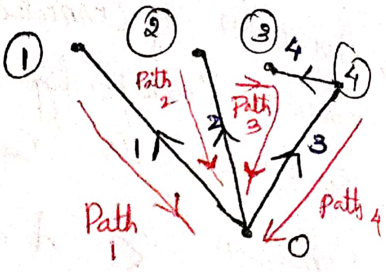
2.2 Branch-path incidence matrix 'K'

* The incidence of branches to paths in a tree is shown by the branch-path incidence matrix, where a path is oriented from bus to reference node.

* Elements of the matrix are:
  - $\boxed{K_{ij} = 1}$ → If the i^{th} branch is in the path from j^{th} bus to reference & is oriented in same direction.
  - $\boxed{K_{ij} = -1}$ → If the i^{th} branch is in the path from j^{th} bus to reference but oriented in opposite direction.
  - $K_{ij} = 0$ → If the i^{th} branch is not in the path from j^{th} bus to reference.

* $n = 5$, $b = n-1 = 5-1 = 4$. Where b–branch.

$$K = \begin{array}{c|cccc} b \backslash \text{Path} & ① & ② & ③ & ④ \\ \hline 1 & -1 & \bullet & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & -1 \\ 4 & 0 & 0 & -1 & 0 \end{array}$$



* Branch-path incidence matrix & Submatrix A_b relates the branches to paths & branches to buses resp.

$$\therefore \quad A_b \cdot K^T = U$$

$$\therefore \quad \boxed{K^T = [A_b]^{-1}}$$

2.3 Basic Cutset Incidence Matrix 'B'

* The incidence of elements to basic cut sets of a connected graph is shown by the basic cut-set incidence matrix 'B'.
* The elements of the matrix are:

$\boxed{b_{ij} = 1}$ → If i^{th} element is incident to & oriented in the same direction as the j^{th} basic cut-set

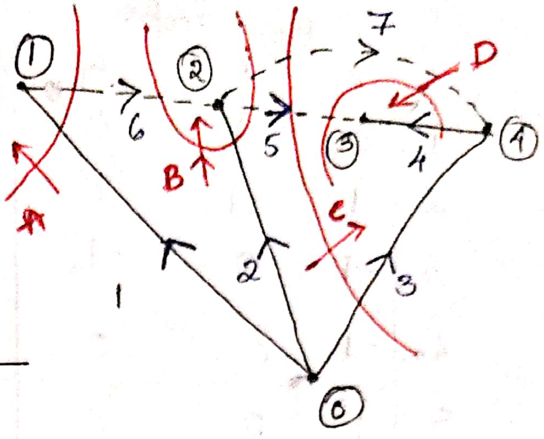
$\boxed{b_{ij} = -1}$ → If i^{th} element is incident to & oriented in opposite direction as the j^{th} basic cut set

$\boxed{b_{ij} = 0}$ → If i^{th} element is not incident to j^{th} basic cut set.

* Basic cut-set incidence matrix dimension is $B = e \times b$.
* Consider the cut-set graph shown previously.

Basic cut-sets

$$B = \begin{array}{c|c} e \backslash b & \begin{matrix} A & B & C & D \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \end{array} = \begin{array}{c|c} e \backslash b & \text{Basic cut set} \\ \hline \begin{matrix} \text{Branches} \\ \text{Links} \end{matrix} & \begin{bmatrix} U_B \\ B_l \end{bmatrix} \end{array}$$



Here $U_B \rightarrow$ Identity Matrix $\rightarrow$ Corresponds to Branches & cut sets.

* $B_l \rightarrow$ Can be obtained from Bus incidence Matrix as follows.

$$B_l \cdot A_b = A_l$$

$$B_l = A_l \cdot A_b^{-1}$$

W.K.T. $A_b^{-1} = K^T$

$\therefore \boxed{B_l = A_l \cdot K^T} \rightarrow$ B_l in terms of Bus incidence matrix.

2.4: Basic loop incidence Matrix 'C'

* The incidence of elements to basic loops of connected graph is shown by basic loop incidence matrix C
* The elements of the matrix are:

  $C_{ij} = 1 \rightarrow$ If the i^{th} element is incident to & oriented in the same direction as j^{th} basic loop.

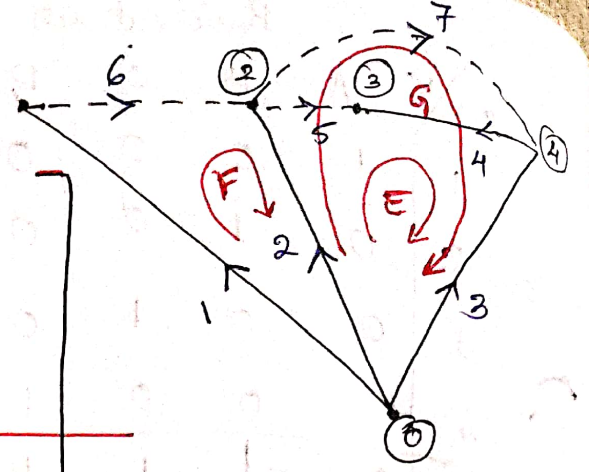
  $C_{ij} = -1 \rightarrow$ If the i^{th} element is incident to & oriented in opposite direction as j^{th} basic loop.

  $C_{ij} = 0 \rightarrow$ If the i^{th} element is not incident to j^{th} basic loop

* The basic loop incidence matrix dimension is $C = e \times l$.

Basic loops

$$C = \begin{array}{c|ccc} e \backslash l & E & F & G \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & 0 & -1 \\ 4 & -1 & 0 & 0 \\ \hline 5 & 1 & 0 & 0 \\ 6 & 0 & 1 & 0 \\ 7 & 0 & 0 & 1 \end{array} = \begin{array}{c|c} e \backslash l & \text{Basic loop} \\ \hline \text{Branches} & C_b \\ \hline \text{Links} & U_l \end{array}$$



Where U_l - Identity Matrix

3. Primitive Network:

* An element in an electrical n/w is completely characterised by the relationship b/w the current through the element & Vtg a/c it.
* A primitive element is a fundamental element which is not connected to any other element. A set of such unconnected elements is defined as a primitive network.
* Primitive n/w components can be represented in both impedance form & admittance form.
* Let,

  V_{pq} - Vtg a/c the p-q

  e_{pq} - Source Vtg in series with element p-q

  i_{pq} - Current through element p-q

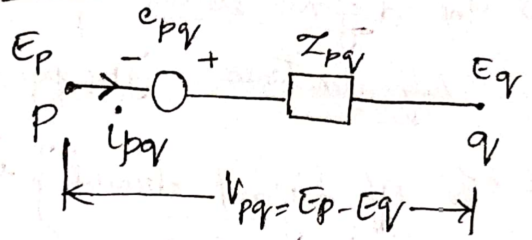
  j_{pq} - Source Current in parallel with element p-q

  Z_{pq} - Self impedance of element p-q

  y_{pq} - Self admittance of element p-q.

* Each element has 2 Variables, V_{pq} & i_{pq}. In steady state, these Variables & the parameters of the elements Z_{pq} & Y_{pq} – real nos for dc ckt
Z_{pq} & Y_{pq} – are Complex nos. for ac ckts.

(i) primitive n/w components in impedance form:



* The performance eqn of an element in impedance form is

$$\boxed{V_{pq}+e_{pq}=Z_{pq}\cdot i_{pq}}$$

* Expressing the Variables as Vectors & Parameters as matrices, the Performance Eqn in impedance form is,

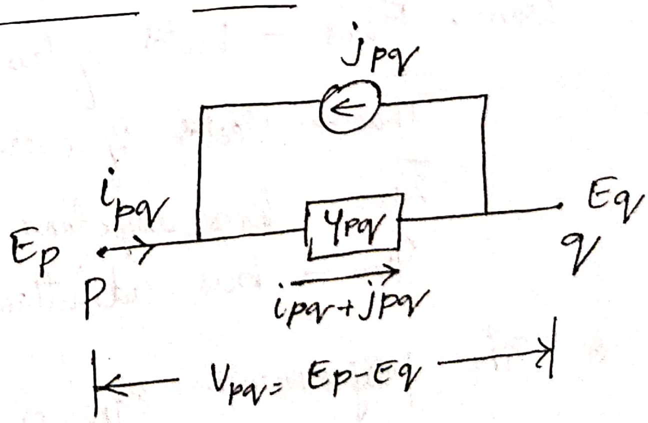
$$\boxed{\bar{V}+\bar{e}=[Z]\vec{i}}$$

* The diagonal element of the matrix $[Z]$ of primitive n/w is Self impedance $Z_{pq,pq}$
* Off diagonal element b/w p-q & r-s is → Mutual impedance $Z_{pq,rs}$
* The matrix $[Z]$ is a diagonal matrix if there is no mutual Coupling b/w elements. In this case Self impedances are equal to the reciprocals of corresponding self admittances.

(ii) Primitive n/w Components in admittance form:

* The performance eqn of an element in admittance form is

$$\boxed{i_{pq}+j_{pq}=Y_{pq}\cdot V_{pq}}$$



* The parallel source current in admittance form is related to series source vtg in impedance form by

$$\boxed{j_{pq}=-Y_{pq}\cdot e_{pq}}$$

* expressing variables as vectors & parameters as matrices, the performance eqn in admittance form is

$$\boxed{\vec{i} + \vec{j} = [y]\,\bar{v}.}$$

* The diagonal element of the matrix $[y]$ of the primitive n/w is self admittance $y_{pq,pq}$.
* The off diagonal element} is → Mutual admittance $y_{pq,rs}$ b/w p-q & r-s
* The matrix $[y]$ is a diagonal matrix if there is no mutual coupling b/w elements. In this case, the self-impedances are equal to the reciprocals of corresponding self admittances
* The primitive admittance matrix $[y]$ can be obtained by inverting the primitive impedance matrix $[z]$

4. Formation of Ybus by Singular Transformation method:

* The performance equations relating the bus voltages to bus current injections are given by

$$\bar{E}_{bus} = Z_{bus} \cdot \bar{I}_{bus} \rightarrow (1)$$

$$\bar{I}_{bus} = Y_{bus} \cdot \bar{E}_{bus}. \rightarrow (2)$$

Where, $\bar{E}_{bus}$ – Vector of bus voltages measured wrt reference bus

$\bar{I}_{bus}$ – Vector of currents injected into the bus.

Z_{bus} – bus impedance matrix

Y_{bus} – bus admittance matrix

* The performance Eqn of primitive n/w in admittance form is,

$$\bar{i} + \bar{j} = [y].\bar{v}. \longrightarrow (3).$$

Where

Premultiplying ③ by A^T

$$A^T[\bar{i}] + A^T[\bar{j}] = A^T[Y].[\bar{V}] \rightarrow ④$$

From KCL, Sum of currents at junction is equal to 0.

$$\therefore A^T[\bar{i}] = 0 \longrightarrow ⑤$$

$A^T[\bar{j}] \rightarrow$ is the algebraic sum of source currents @ each bus equal to vector of impressed bus current.

$$\therefore A^T[\bar{j}] = \bar{I}_{bus} \longrightarrow ⑥$$

Substitute ⑤ & ⑥ in ④, $\bar{I}_{bus} = A^T[Y][\bar{V}] \rightarrow ⑦$

The power into the n/w is,

$$[\bar{I}_{bus}^*]^T.[\bar{E}_{bus}]$$

Sum of power in primitive n/w is,

$$[\bar{j}^*]^T.\bar{V}$$

Power in primitive N/W = power in the interconnected N/W.

$$[\bar{I}_{bus}^*]^T.[\bar{E}_{bus}] = [\bar{j}^*]^T.\bar{V} \longrightarrow ⑧$$

Taking the conjugate transpose of ⑥

$$[(A^T)^*]^T.[\bar{j}^*]^T = [\bar{I}_{bus}^*]^T \rightarrow ⑨$$

Substitute ⑨ in ⑧,

$$[\bar{E}_{bus}] \times [(A^T)^*]^T.\cancel{[\bar{j}^*]^T} = \cancel{[\bar{j}^*]^T}[\bar{V}]$$

$$[\bar{E}_{bus}].A^* = [\bar{V}] \rightarrow ⑩$$

$[A]$ is a real matrix, hence $A^* = A$

⑩ $\Rightarrow$ $[A][\bar{E}_{bus}] = [\bar{V}] \rightarrow ⑪$ ⑪

Premultiplying ③ by A^T

$$A^T[\bar{i}] + A^T[\bar{j}] = A^T[Y].[\bar{V}] \rightarrow ④$$

From KCL, Sum of currents at junction is equal to 0.

$$\therefore \quad A^T[\bar{i}] = 0 \longrightarrow ⑤$$

$A^T[\bar{j}] \rightarrow$ is the algebraic sum of source currents @ each bus equal to vector of impressed bus current.

$$\therefore \quad A^T[\bar{j}] = \bar{I}_{bus} \longrightarrow ⑥$$

Substitute ⑤ & ⑥ in ④,

$$\bar{I}_{bus} = A^T[Y][\bar{V}] \rightarrow ⑦$$

The power into the n/w is,

$$[\bar{I}_{bus}^*]^T.[\bar{E}_{bus}]$$

Sum of power in primitive n/w is,

$$[\bar{j}^*]^T.\bar{V}$$

Power in primitive N/W = power in the interconnected N/W.

$$[\bar{I}_{bus}^*]^T.[\bar{E}_{bus}] = [\bar{j}^*]^T.\bar{V} \longrightarrow ⑧$$

Taking the conjugate transpose of ⑥

$$[(A^T)^*]^T.[\bar{j}^*]^T = [\bar{I}_{bus}^*]^T \rightarrow ⑨$$

Substitute ⑨ in ⑧,

$$\cancel{[\bar{E}_{bus}] \times [(A^T)^*]^T.[\bar{j}^*]^T = [\bar{j}^*]^T[\bar{V}]}$$

$$[\bar{E}_{bus}].A^* = [\bar{V}] \rightarrow ⑩$$

$[A]$ is a real matrix, hence $A^* = A$

$$⑩ \Rightarrow \quad [A].[\bar{E}_{bus}] = [\bar{V}] \rightarrow ⑪$$

⑪

The performance Eqn can be obtained by Eqn 7 & 11

i.e $[\bar{E}_{bus}] \times [A] = [\bar{V}]$

$\bar{I}_{bus} = A^T [Y][\bar{V}]$.

Combining the above 2 Eqns,

$$\bar{I}_{bus} = A^T [Y] . [\bar{E}_{bus}] \times [A]$$

Re-arranging above eqn : $\dfrac{\bar{I}_{bus}}{\bar{E}_{bus}} = A^T [Y] . A$

From ohm's law: $V = IZ \Rightarrow$

$$\therefore \boxed{Y_{bus} = A^T [Y] . A}$$

5. Formation of Ybus by Inspection method:

* Let 'N' be the no. of major nodes in a n/w.
* Let, $V_1, V_2, V_3, \ldots V_n \rightarrow$ Node Voltages of nodes $1, 2, 3, \ldots, n$ resp.

  $I_{11}, I_{22}, I_{33}, \ldots I_{nn} \rightarrow$ Sum of current sources connected to nodes $1, 2, 3, \ldots n$ resp.

Y_{ii} = Sum of admittances connected to node-i

Y_{ij} = Negative of sum of admittances connected b/w nodes i & j.

* For N bus s/s with n no. of nodal Eqns, we have.

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + \cdots + Y_{1n}V_n = I_{11}$$
$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + \cdots + Y_{2n}V_n = I_{22}$$
$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + \cdots + Y_{3n}V_n = I_{33} \rightarrow (1)$$
$$\vdots$$
$$Y_{n1}V_1 + Y_{n2}V_2 + Y_{n3}V_3 + \cdots + Y_{nn}V_n = I_{nn}$$

Eqn (1) in matrix form,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3n} \\ \vdots & \vdots & & & \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \\ \vdots \\ I_{nn} \end{bmatrix} \rightarrow (2)$$

$$(2) \Rightarrow \quad YV = I \rightarrow (3)$$

In power s/s, Y matrix is designated as Ybus, → Bus admittance ~~Bus adm~~
Nodes Vtgs → Bus Voltages.

∴ Eqn (3) can be written as, $\boxed{Y_{bus} \cdot V = I}$

where, Y_{bus} – symmetrical around principal diagonal.

$Y_{11}, Y_{22}, Y_{33}, \cdots Y_{nn}$ → Self admittances → Diagonal elements

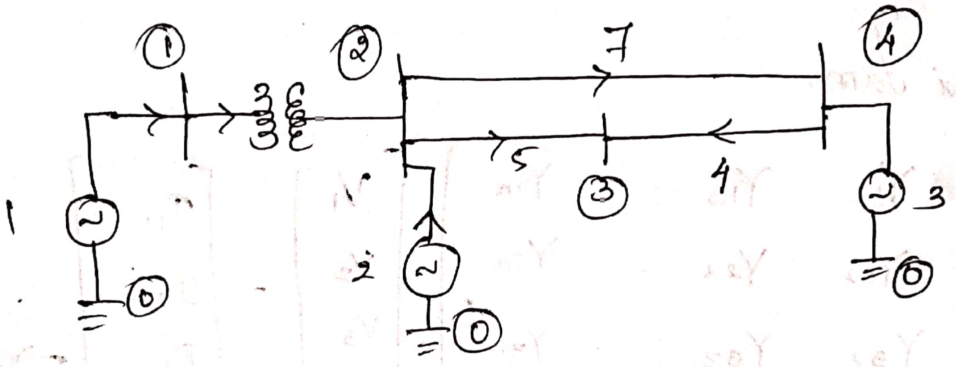
other Elements → Mutual admittances → off diagonal elements

6.1 Problems of Elementary graph Theory:

8M

1. For the n/w shown, formulate the oriented graph, tree, bus incidence matrix (A), branch-path incidence matrix (K) Basic cut set incidence matrix (B), basic loop incidence matrix (C) Choose elements (1, 2, 3, 4) as tree-branches:

J-24



Soln: Solved in concepts 1.3

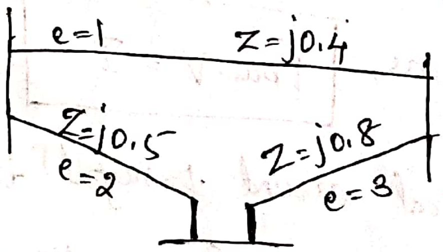
2. Obtain the primitive impedance matrix & primitive admittance matrix using the data shown below.

6M

J-24

$[Z] \rightarrow$ impedances (self) of elements (e)

$Z_m \rightarrow$ mutual impedances (in pu) b/w e=1 & e=2, $Z_m = 0.1j$ pu.



Soln, To find $[Z_{prim}]$ & $[Y_{prim}]$

For above n/w, $n = 3$, $\therefore$ $[Z_{prim}] = \begin{bmatrix} Z_1 & Z_m & 0 \\ Z_m & Z_2 & 0 \\ 0 & 0 & Z_3 \end{bmatrix}$

Here $Z_{prim} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \Rightarrow$

Here: Diagonal elements $Z_{11} = Z_1$, $Z_{22} = Z_2$, $Z_{33} = Z_3$

off diagonal element $Z_{12} = Z_{21} = Z_m$

$\therefore [Z_{prim}] = \begin{bmatrix} j0.4 & j0.1 & 0 \\ j0.1 & j0.5 & 0 \\ 0 & 0 & j0.8 \end{bmatrix}$

Primitive admittance matrix $Y_{prim} = [Z_{prim}]^{-1}$

$$Y_{prim} = \begin{bmatrix} j0.4 & j0.1 & 0 \\ j0.1 & j0.5 & 0 \\ 0 & 0 & j0.8 \end{bmatrix}^{-1} \quad ; \quad \text{WKT} \quad [X]^{-1} = \frac{1}{|X|} \cdot \text{adj}[X]$$

$$|X| = j(0.4)[j0.5 \times j0.8] - j0.1[j0.1 \times j0.8] + 0 = -0.152j$$

$$\text{adj}[X] = \begin{bmatrix} +(-0.4) & -(-0.08) & +0 \\ -(-0.08) & +(-0.32) & -0 \\ +0 & -0 & +(-0.19) \end{bmatrix} = \begin{bmatrix} -0.4 & 0.08 & 0 \\ 0.08 & -0.32 & 0 \\ 0 & 0 & -0.19 \end{bmatrix}$$

$$Y_{prim} = \frac{1}{|X|} \cdot \text{adj}[X] = \frac{1}{-0.152j} \times \begin{bmatrix} -0.4 & 0.08 & 0 \\ 0.08 & -0.32 & 0 \\ 0 & 0 & -0.19 \end{bmatrix} = \begin{bmatrix} -j2.63 & -j0.526 & 0 \\ j0.5263 & -j2.105 & 0 \\ 0 & 0 & -j1.25 \end{bmatrix}$$

3) 6M. What is primitive n/w? obtain primitive impedance matrix & primitive admittance matrix for the elements given in table below. JL-23

Element no.	Self impedance $Z_{pq.pq}$		Mutual impedance $Z_{pq.rs}$	
	Bus code p-q	Impedance in pu	Bus code r-s	Impedance in pu
1	1-2	j0.2	–	–
2	2-3	j0.4	1-2	j0.1
3	1-3	j0.3	2-3	j0.1

Soln: $n = 3$, $\therefore$ $Z_{prim} = n \times n = 3 \times 3$; $Z_{prim} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$

$$Z_{prim} = \begin{bmatrix} j0.2 & j0.1 & 0 \\ j0.1 & j0.4 & j0.1 \\ 0 & j0.1 & j0.3 \end{bmatrix}$$

Given: $Z_{11} = j0.2$ $\rightarrow$ diagonal
$Z_{22} = j0.4$
$Z_{33} = j0.3$

$\underset{= Z_{21}}{Z_{12}}$ & $\underset{= Z_{32}}{Z_{23}}$ = $j0.1$ $\rightarrow$ off diagonal

WKT $Y_{prim} = [Z_{prim}]^{-1} = \begin{bmatrix} j0.2 & j0.1 & 0 \\ j0.1 & j0.4 & j0.1 \\ 0 & j0.1 & j0.3 \end{bmatrix}^{-1}$; WKT $[X]^{-1} = \frac{1}{|X|} \cdot adj[X]$

$|X| = j(0.2)[(j0.4 \times j0.3) - (j0.1 \times j0.1)] - j0.1(j0.1 \times j0.3 - 0) + 0$

$|X| = \underline{\underline{-0.019j}}$

$$adj[X] = \begin{bmatrix} +(-0.11) & -(-0.03) & +(-0.01) \\ -(-0.03) & +(-0.06) & -(-0.02) \\ +(-0.01) & -(-0.02) & +(-0.07) \end{bmatrix}$$

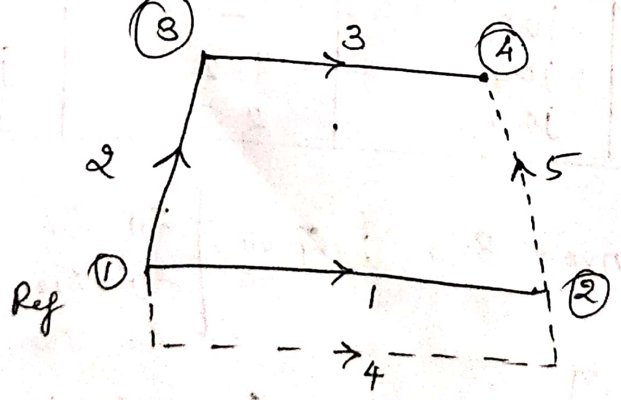
$$[X]^{-1} = \frac{1}{|X|} \cdot adj[X] = \frac{1}{-0.019j} \times \begin{bmatrix} -0.11 & 0.03 & -0.01 \\ 0.03 & -0.06 & 0.02 \\ -0.01 & 0.02 & -0.07 \end{bmatrix}$$

$$Y_{prim} = \begin{bmatrix} -5.78j & 1.57j & -0.526j \\ 1.57j & -3.157j & 1.052j \\ -0.526j & 1.052j & -3.684j \end{bmatrix}$$

7) Form the incidence matrices A, B, C for the graph shown & prove that $B'C = 0$. Select 1, 2, 3 as tree, node ① as reference.

8M

JL-23



Soln: From above graph: $n = 4$

$e = 5$

$b = n - 1 = 3$

$l = e - n = 5 - 3 = 2$

* Element node incidence matrix : $\hat{A}$

$$\hat{A} = \begin{array}{c|c} e \backslash n & \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \end{array} \Rightarrow$$

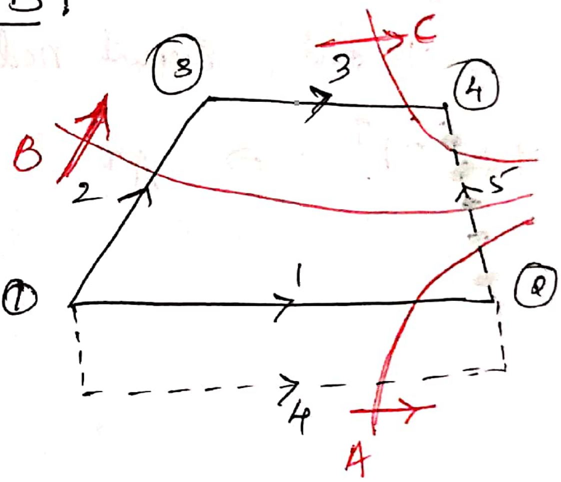
Node ① → reference node, we get

$$A = \begin{array}{c|c} e \backslash \text{bus} & \begin{array}{ccc} ② & ③ & ④ \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \end{array}$$

* Basic cut-set incidence matrix – B :

→ Basic cut sets

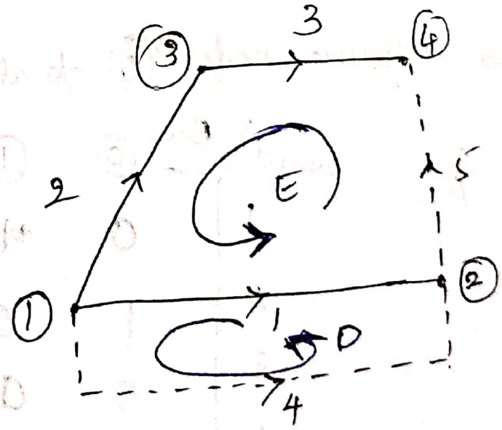
$$B = \begin{array}{c|c} e \backslash b & \begin{array}{ccc} Ⓐ & Ⓑ & Ⓒ \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} \end{array}$$



* Basic loop incidence matrix – C :

→ loops

$$C = \begin{array}{c|c} e \backslash l & \begin{array}{cc} D & E \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$



5. The bus incidence matrix of a power system is shown below.

6M

Jl-23

$$A = \begin{array}{c|c} \text{Buses} \backslash \text{elements} & \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

Obtain element node incidence matrix. Hence draw the oriented graph.

Soln: Given: Bus incidence Matrix - A

To find Element node incidence matrix $\hat{A}$.

Take A^T $\therefore \Rightarrow$

$$A^T = \begin{array}{c|cccc} \text{Element} \backslash \text{Bus} & (1) & (2) & (3) & (4) \\ \hline \begin{array}{c}1\\2\\3\\4\\5\\6\\7\end{array} & \multicolumn{4}{c}{\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \end{bmatrix}} \end{array}$$

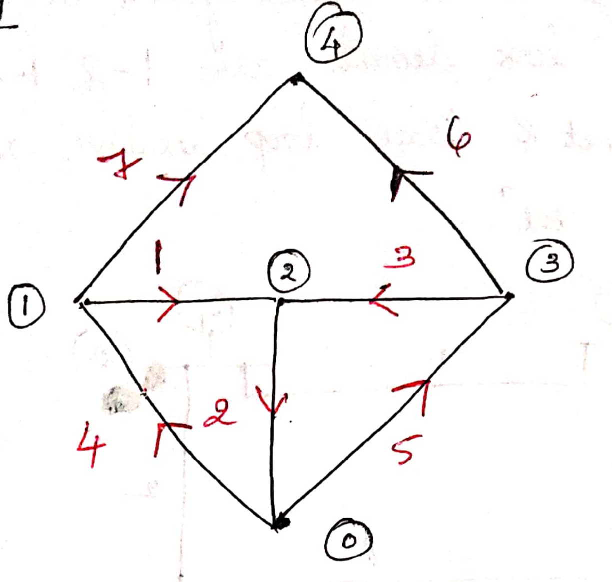
Add reference node (0) to above matrix to get $\hat{A}$

$$\therefore \hat{A} = \begin{array}{c|ccccc} \text{element} \backslash \text{Bus} & (0) & (1) & (2) & (3) & (4) \\ \hline \begin{array}{c}1\\2\\3\\4\\5\\6\\7\end{array} & \multicolumn{5}{c}{\begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}} \end{array}$$

~~Note~~

Note: Column (0) i.e. Bus (0) is filled considering the summation of each row to be zero.

Oriented graph of $\hat{A}$:-



6. The bus incidence matrix A for a n/w of 8 elements & 5 nodes (4 buses) is given in table below. Reconstruct the oriented graph. Hence obtain the one-line diagram of the s/s indicating the generator position.

6M J-23

$$A = \begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & -1 \\ 3 & 0 & 0 & 1 & -1 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}$$

Soln:- Step 1 Given bus incidence matrix - A.

Step 2 * To find element node incidence matrix $\hat{A}$ to find oriented graph

Step 3 Take A^T ⇒

$$A^T = \begin{array}{c|cccc} \text{element} \backslash \text{node} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & -1 & 1 & 0 & 0 \\ 6 & 0 & -1 & 1 & 0 \\ 7 & -1 & 0 & 0 & 1 \\ 8 & 0 & -1 & 0 & 1 \end{array}$$

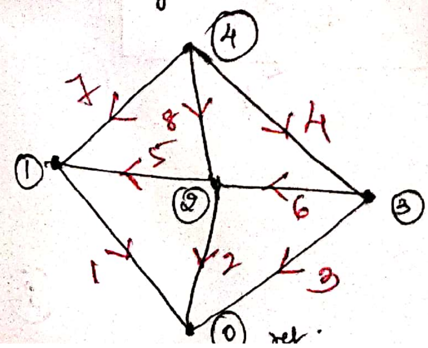
Step 4

$$\Rightarrow \hat{A} = \begin{array}{c|ccccc} e \backslash \text{node} & 0 & 1 & 2 & 3 & 4 \\ \hline 1 & -1 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & -1 & 1 & 0 & 0 \\ 6 & 0 & 0 & -1 & 1 & 0 \\ 7 & 0 & -1 & 0 & 0 & 1 \\ 8 & 0 & 0 & -1 & 0 & 1 \end{array}$$

Oriented graph.

Step 5

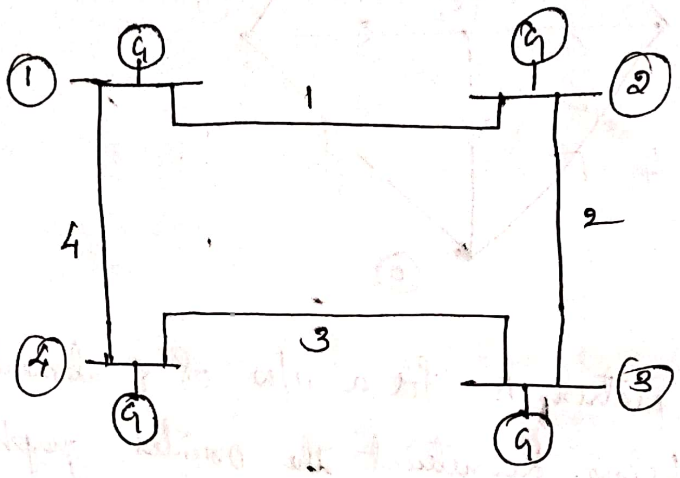
Oriend graph



(7) For the power s/s shown, select ground as reference & a tree for which link elements are 1-2, 1-4, 2-3, 3-4. Obtain basic cut set & basic loop incidence matrices. Verify the relation $C_b = -B_l^T$.

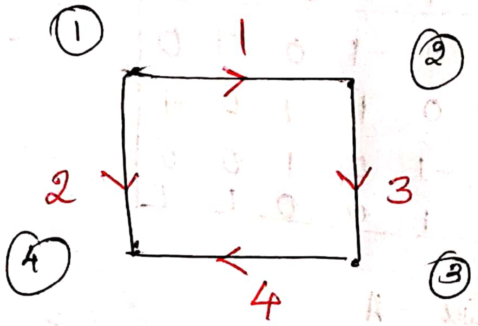
7M

Jl-22



Soln: Soln:

Convert the given n/w to graph.



Here $e = 4$

$n = 4$

links 'l' $= e - n + 1$

$l = 4 - 4 + 1$

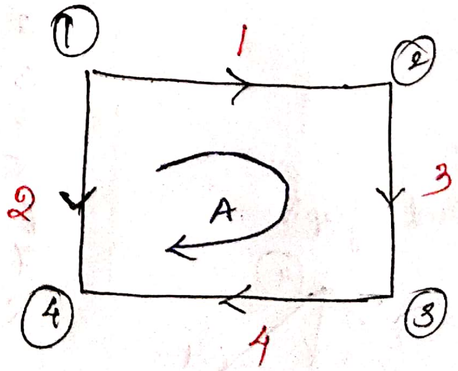
$l = 1$

No. of loops = no. of links $= 1$ $\longrightarrow$ (element : 4)

* last element in graph is always link

No. of basic cut set $= n - 1 = 4 - 1 = 3$

Basic loop incidence matrix : 'C'

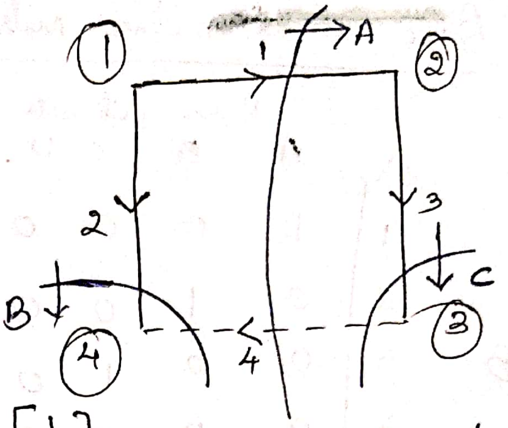


$$C = \begin{array}{c|c} e & \text{loops } A \\ \hline 1 & 1 \\ 2 & -1 \\ 3 & 1 \\ \hline 4 & 1 \end{array} = \begin{bmatrix} C_b \\ \hline U_l \end{bmatrix}$$

Basic cut set - incidence matrix 'B':

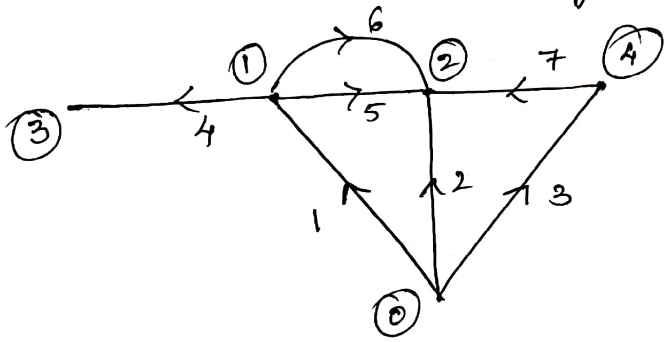
$$B = \begin{array}{c|ccc} e & A & B & C \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ \hline 4 & -1 & 1 & -1 \end{array} = \begin{bmatrix} U_B \\ B_l \end{bmatrix}$$

(Cut sets: A, B, C)



Take $[B_l]^T = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ → Take $-[B_l]^T = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = C_b$ → Hence proved.

8. The Oriented Connected graph of a s/s is shown. Obtain basic cutset & basic loop incidence matrices. Hence verify the relation (i) $B_l = A_{lk}^T$ (ii) $B^T C = 0$. Take ground as reference.



Soln: Here

$n = 5$

$e = 7$

links $= l = e - n + 1 = 7 - 5 + 1 = \underline{\underline{3}}$

(elements ; 5, 6, 7) ★ — last elements in graph.

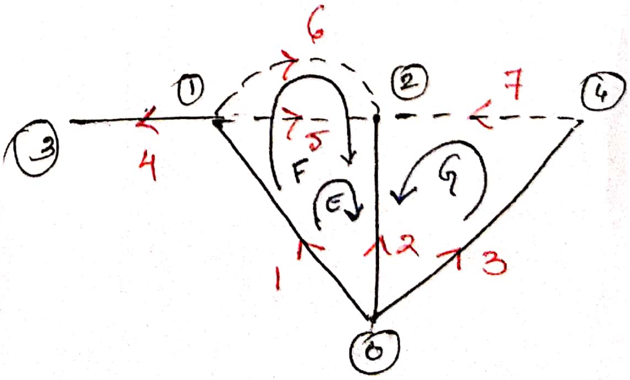
No. of loops = No. of links = 3.

No of basic cut set $= n - 1 = 5 - 1 = 4$.

Basic loop incidence matrix 'C':-

$$C = \begin{array}{c|ccc} e & E & F & G \\ \hline 1 & 1 & 1 & 0 \\ 2 & -1 & -1 & -1 \\ 3 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 \\ \hline 5 & 1 & 0 & 0 \\ 6 & 0 & 1 & 0 \\ 7 & 0 & 0 & 1 \end{array} = \begin{bmatrix} C_b \\ U_l \end{bmatrix}$$

(Loops: E, F, G)

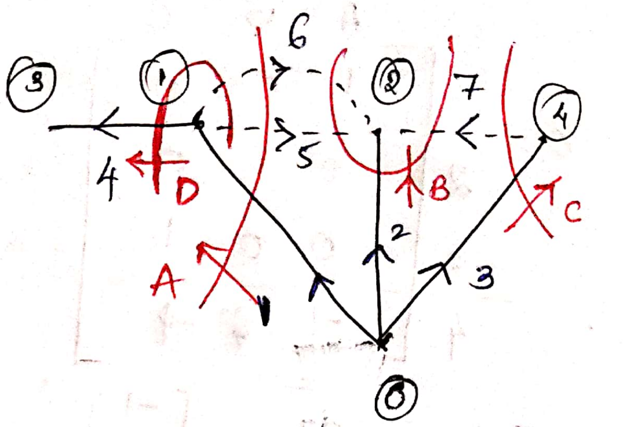


loop orientation is same as link orientation

Basic Cut Set Incidence Matrix 'B':

Basic cut sets

$$B = \begin{array}{c|cccc} e & A & B & C & D \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ \hline 5 & -1 & 1 & 0 & -1 \\ 6 & -1 & 0 & 0 & -1 \\ 7 & 0 & 1 & -1 & 0 \end{array} = \begin{bmatrix} U_B \\ B_l \end{bmatrix}$$



Cut set - Direction is same as branch / twigs